RESOLUTION AND ACCURACY ESTIMATES FOR NINE AXIS ACCELEROMETER CONFIGURATION

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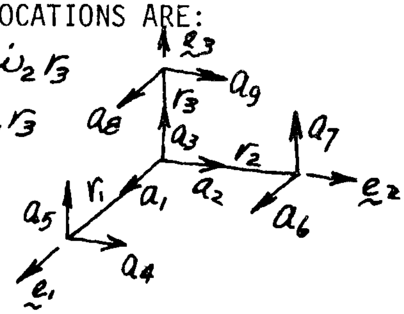
THE ACCELERATIONS AT THE FOUR ACCELEROMETER LOCATIONS ARE:

$$
\begin{aligned}
a_1 &= \ddot{x}_1 & a_8 &= \ddot{x}_1 + \omega_1\omega_3 r_3 + \dot{\omega}_2 r_3 \\
a_2 &= \ddot{x}_2 & a_9 &= \ddot{x}_2 + \omega_2\omega_3 r_3 - \dot{\omega}_1 r_3 \\
a_3 &= \ddot{x}_3 \\
a_4 &= \ddot{x}_2 + \omega_1\omega_2 r_1 + \dot{\omega}_3 r_1 \\
a_5 &= \ddot{x}_3 + \omega_1\omega_3 r_1 - \dot{\omega}_2 r_1 \\
a_6 &= \ddot{x}_1 + \omega_1\omega_2 r_2 - \dot{\omega}_3 r_2 \\
a_7 &= \ddot{x}_3 + \omega_2\omega_3 r_2 + \dot{\omega}_1 r_2
\end{aligned}
$$



where a_i . . . Translational accel. along sensitive axis of i^{th} accel.

$\ddot{x}_j$ . . . j^{th} component of translational accel. of rigid body

ω_j . . . j^{th} " " angular velocity "

$\dot{\omega}_j$ . . . j^{th} " " " acceleration "

r_j . . . j^{th} separation arm

These are the Forward Equations. They are identical with King, et al (3a,b & c) applied to each accelerometer. King solves these equations for $\dot{\omega}_j$ (13-15). These are the Inverse Equations, and in this notation they are

$$
\begin{aligned}
\dot{\omega}_1 &= (a_7 - a_3)/2r_2 - (a_9 - a_2)/2r_3 \\
\dot{\omega}_2 &= (a_8 - a_1)/2r_3 - (a_5 - a_3)/2r_1 \\
\dot{\omega}_3 &= (a_4 - a_2)/2r_1 - (a_6 - a_1)/2r_2
\end{aligned}
$$

These Equations can be written in compact form via matrix notation:

$$
\underline{\dot{\omega}} = \frac{1}{2} C \underline{a} \quad \text{where} \quad \underline{\dot{\omega}} = \begin{Bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{Bmatrix}, \quad \underline{a} = \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{Bmatrix}
$$

and the matrix C is

$$C=\begin{bmatrix} 0 & 1/r_3 & -1/r_2 & 0 & 0 & 0 & 1/r_2 & 0 & -1/r_3 \\ -1/r_3 & 0 & 1/r_1 & 0 & -1/r_1 & 0 & 0 & 1/r_3 & 0 \\ 1/r_2 & -1/r_1 & 0 & 1/r_1 & 0 & -1/r_1 & 0 & 0 & 0 \end{bmatrix}$$

It is important to note that the Inverse Equations are Linear. The Forward Equations are nonlinear, sinch they contain cross-products in angular velocities ($\underline{\omega}$). It is the linearity of the Inverse Equations that make them very useful for computation.

According to the method of King et al, accelerometers are placed at the nine locations in the sketch. Let the calibrated response of these accelerometers be

$$a^* = \begin{Bmatrix} a_1^* \\ a_2^* \\ \vdots \\ a_9^* \end{Bmatrix}$$

RESOLUTION:

Suppose the accelerometers are perfect, i.e., $\underline{a}^* = \underline{a}$.
What is the change in the angular acceleration produced by the smallest perceptible change in the accelerometer output, i.e. The Resolution in angular accel.? Let this change in output be $\underline{\delta}$ so that

$$\underline{a}^* = \underline{a} + \underline{\delta}$$

and the Inverse Equations yield

$$\underline{\dot{\omega}}^* = \tfrac{1}{2}C\underline{a}^* = \tfrac{1}{2}C(\underline{a}+\underline{\delta})$$

Since the Equations are Linear, $\underline{\dot{\omega}} = \frac{1}{2}C\underline{a}$ and the change in angular acceleration is

$$\Delta\underline{\dot{\omega}} = \tfrac{1}{2}C\underline{\delta}$$

The smallest possible change would be to allow one accelerometer to change one digital unit. Call this unit α, and consider a change α in accelerometer a_1:

$$\underline{\delta} = \begin{Bmatrix} \alpha \\ 0 \\ \vdots \\ 0 \end{Bmatrix}, \quad \Delta\underline{\dot{\omega}} = \tfrac{1}{2}C\underline{\delta} = \begin{Bmatrix} 0 \\ -\frac{\alpha}{2r_3} \\ \frac{\alpha}{2r_2} \end{Bmatrix} = \begin{Bmatrix} \Delta\dot{\omega}_1 \\ \Delta\dot{\omega}_2 \\ \Delta\dot{\omega}_3 \end{Bmatrix}$$

Thus the general form for the Resolution is

$$\Delta\dot{\omega}_j = \frac{\alpha}{2r_i}$$

Where the i & j indices depend on which component of angular acceleration and which accelerometer is in question.

The current design value of α is .366 g's = 142 in/sec^2

Based on this value of α, the Resolution for various separation arms is given in the following table:

r (in)	$\Delta\dot{\omega}$ (RAD/SEC^2)
.3	236
.4	178
.5	142
.7	101
1.0	71

ACCURACY:

The translational accelerometers have a certain error associated with them which is usually expressed as a percentage of the full scale reading. The accelerometer output is

$$\underline{a}^* = \underline{a} + \underline{\beta}$$

where $\underline{\beta}$ is the error signal.

How do these errors $\underline{\beta}$ effect the accuracy of the calculated angular accelerations? The Inverse Equations $\Rightarrow$

$$\underline{\dot{\omega}}^* = \frac{1}{2}C\underline{a}^* = \frac{1}{2}C(\underline{a}+\underline{\beta}) = \frac{1}{2}C\underline{a} + \frac{1}{2}C\underline{\beta}$$

But the exact (no error) result is

$$\underline{\dot{\omega}} = \frac{1}{2}C\underline{a}$$

so

$$\Delta\underline{\dot{\omega}} = \underline{\dot{\omega}}^* - \underline{\dot{\omega}} = \frac{1}{2}C\underline{\beta}$$

What is the maximum error in angular accel. ($\Delta\underline{\dot{\omega}}$) that can be caused by an error in translational acceleration $\underline{\beta}$? The answer is given by the Chebyshev Norm and a theorem from matrix alegbra. This result in matrix notation is

$$\| \Delta\underline{\dot{\omega}} \| \leq \frac{1}{2} \| C \| \, \| \beta \|$$

or in component form

$$\max_j | \Delta\dot{\omega}_j | \leq \frac{1}{2}\left[\max_j \sum_i C_{ji} \right] \left(\max_j |\beta_j| \right)$$

The term in brackets is called the Chebyshev Norm of C. For equally spaced accelerometers ($r_1 = r_2 = r_3 = r$),

$$\| C \| = \frac{4}{r}$$

Thus the maximum error in angular acceleration is

$$\Delta\dot{\omega}\Big|_{Max} \leq \left(\tfrac{1}{2}\right)\frac{4}{r}\beta\Big|_{Max} = \frac{2}{r}\beta\Big|_{Max}$$

A good translational accelerometer has an accuracy of 1% full scale. So

$$\beta\Big|_{Max} = (.01)(150\,g\text{'s})(386\ \text{in sec}^{-2}g^{-1}) = 579\ \text{in/sec}^2$$

Here is a table based on this value of max:

r (in)	$\Delta\dot{\omega}\big|_{max}$ (rad/sec^2)
.3	3860
.4	2895
.5	2316
.7	1654
1.0	1158

This is a conservative estimate of the accuracy. It represents an upper bound on the error, i.e., a worst case analysis. However, it is a straight forward and mathematically correct approach to the question of accuracy.

How can we get a better feel for the errors associated with a typical experiment? It would be possible to numerically simulate an experiment; here is the procedure:

(1) Input: $\dot{\underline{\omega}}(t)$ ... time history of angular acceleration
$\underline{\omega}(t)$ ... " " velocity (which could be calculated from $\dot{\underline{\omega}}(t)$)
$\ddot{\underline{x}}(t)$ ... time history of translational accelerations

(2) System Design: Assume values for
fn ... Natural freq. of accelerometers
j ... Damping ratio "
r_j ... Separation arms

(3) Calculate the accelerations at the nine locations (a_i) via the Forward Equations

(4) Calculate the response of each accelerometers a_i* to the input acceleration a_i from (3) including any error signals.

(5) Apply the Inverse Equations to solve for the Experimental Angular Accelerations, i.e.

$$\dot{\underline{\omega}}^* = 1/2\ C\ \underline{a}^*$$

(6) Compare $\dot{\underline{\omega}}^*$ to $\dot{\underline{\omega}}$ from (1).

Note: This difference will always be $\leq$ the values in the table above.